$$= \frac{8}{\pi^3}\int_0^{\frac{\pi}{2}}\int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi} \varphi(\pi-\varphi)\operatorname{cosec}^2\theta d\theta \sin\varphi\cos\varphi d\varphi$$

$$= \frac{1}{\pi^3}\int_0^{\frac{\pi}{2}}(\pi^2 - 4\theta^2 + 2\theta^2\operatorname{cosec}^2\theta - 4\theta\cot\theta + 2)d\theta$$

$$= \frac{1}{3} + \frac{1}{\pi^2}.$$

QUERY. "In Thompson and Tait's 'Elements of Natural Philosophy' it is stated that, 'at the southern base of a hemispherical hill radius a and density ρ, the true latitude (as measured by the aid of the plumb-line) is diminished by the attraction of the mountain by the angle $\frac{2}{3}\rho\pi a \div (G - \frac{4}{3}\rho a)$, where $G =$ the attraction of the earth in same units.' How is this proved?"

ANSWER BY HENRY HEATON.

As the angle is so small that it is practically equal to its tangent, if $G =$ attraction of the earth, $\frac{2}{3}\rho\pi a =$ horizontal, and $\frac{4}{3}\rho a =$ vertical attraction of the hill, then it is evident that the angle of deflection of the plumb-line $= \frac{2}{3}\rho\pi a \div (G - \frac{4}{3}\rho a)$.

Where the unit of attraction is the attraction of a mass whose volume is 1, density 1, and distance 1, it has been shown, page 194, Vol. II, of the ANALYST, that the horizontal attraction of the hill $= \frac{2}{3}\rho\pi a$; and that the vertical attraction $= \frac{4}{3}\rho a$, I shall now proceed to show.

Taking the origin of co-ordinates at the center of the spherical surface, the attraction exerted by an element of the mass resolved in a vertical direction $= \dfrac{\rho dxdyzdz}{[(x+a)^2 + y^2 + z^2]^{\frac{3}{2}}}$. Hence the vertical attraction of the hill is

$$\rho\int_{-a}^{+a}\int_{-u}^{+u}\int_0^v \frac{dxdyzdz}{[(x+a)^2+y^2+z^2]^{\frac{3}{2}}} = \rho\int_{-a}^{+a}\int_{-u}^{+u}\left[\frac{dxdy}{[(x+a)^2+y^2]^{\frac{1}{2}}}\right.$$

$$\left. - \frac{dxdy}{[2a^2 + 2a]^{\frac{1}{2}}}\right] = \int_{-a}^{+a}\left[\log\left(\frac{\sqrt{(2a)}+\sqrt{(a-x)}}{\sqrt{(2a)}-\sqrt{(a-x)}}\right)dx - \frac{2\sqrt{(a-x)}dx}{\sqrt{(2a)}}\right]$$

$= \frac{4}{3}\rho a$; where the limits of integration, u and v, are, respectively, $u = \sqrt{(a^2 - x^2)}$ and $v = \sqrt{(a^2 - x^2 - y^2)}$.

If we put r for radius of the earth, and δ, its mean density, $G = \frac{4}{3}\delta r$. Substituting this for G in the expression for the angle of deflection, we get, $\rho\pi a \div 2(\delta\pi r - \rho a)$. As a is very small in comparison with r the expression for the deflection is, practically, $\rho a \div 2\delta r$.

[Mr. Adcock obtains precisely the same result as the last above written, viz.; $\rho a \div 2\delta r$.]